Correction du contrôle du 02/04/2014
MMP L2

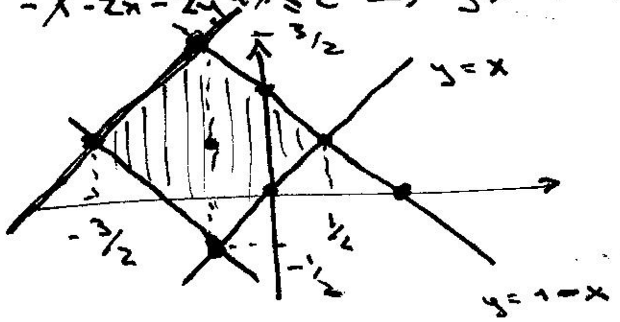
Ex. 1 $D = \{(x,y) : |2x+1| + |2y-1| \leqslant 2\}$

1). $x \geqslant -\frac{1}{2},\ y \geqslant \frac{1}{2} \Rightarrow 2x + \cancel{1} + 2y - \cancel{1} \leqslant 2 \Rightarrow y \leqslant 1 - x$

2). $x \geqslant -\frac{1}{2},\ y \leqslant \frac{1}{2} \Rightarrow 2x+1+1-2y \leqslant 2 \Rightarrow y \geqslant x$

3). $x \leqslant -\frac{1}{2},\ y \geqslant \frac{1}{2} \Rightarrow -1-2x+2y-1 \leqslant 2 \Rightarrow y \leqslant 2+x$

4). $x \leqslant -\frac{1}{2},\ y \leqslant \frac{1}{2} \Rightarrow -\cancel{1}-2x-2y+\cancel{1} \leqslant 2 \Rightarrow y \geqslant -x-1$



$$\iint_D \sin \pi x \, dx\,dy =$$

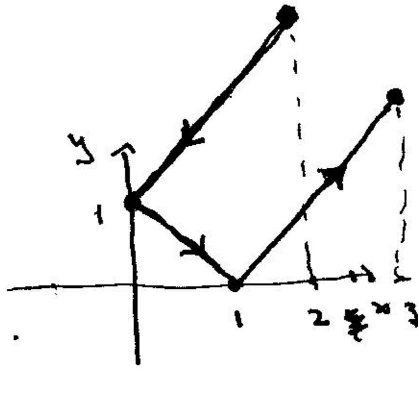
$$= \int_{-3/2}^{-1/2} \left(\int_{-x-1}^{x+2} \sin \pi x \, dy \right) dx + \int_{-1/2}^{1/2} \left(\int_{x}^{1-x} \sin \pi x \, dy \right) dx = \int_{-3/2}^{-1/2} (2x+3) \sin \pi x \, dx$$

$$+ \int_{-1/2}^{1/2} (1-2x) \sin \pi x \, dx = \int_{-1}^{0} (2x+2) \sin \pi\left(x - \tfrac{1}{2}\right) dx + \int_{0}^{1} (2-2x) \sin \pi \left(x - \tfrac{1}{2}\right) dx =$$

$$= -\int_{-1}^{0} \{2(x+1) \cos \pi x\} dx - \int_{0}^{1} 2(1-x) \cos \pi x \, dx = -4 \int_0^1 (1-x) \cos \pi x \, dx =$$

$$= -4 \left[(1-x) \frac{\sin \pi x}{\pi} \right]_0^1 + 4 \left[\frac{\cos \pi x}{\pi^2} \right]_0^1 = -\frac{8}{\pi^2}.$$

Ex. 2 $\begin{cases} x(t) = |t| \\ y(t) = |t-1| \\ t \in [-2, 3] \end{cases}$



$t \in [-2, 0] \Rightarrow \begin{cases} x(t) = -t \\ y(t) = 1 - t \end{cases} \Rightarrow y = x + 1$

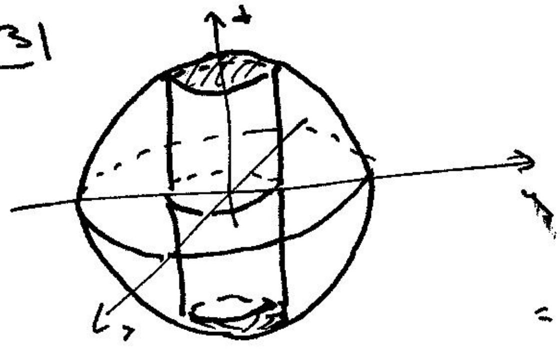
$t \in [0, 1] \Rightarrow \begin{cases} x(t) = t \\ y(t) = 1 - t \end{cases} \Rightarrow y = 1 - x$

$t \in [1, 3] \Rightarrow \begin{cases} x(t) = t \\ y(t) = t - 1 \end{cases} \Rightarrow y = x - 1$

$$\int_\gamma y\,dx + x\,dy = \int_\gamma \vec{\nabla}(xy)\cdot d\vec{r} = (xy)\Big|_{t=-2}^{t=3} =$$

$$= 3\cdot 2 - 2\cdot 3 = 0.$$

Ex. 3|



$\sin\gamma = 1/3$

$$\text{aire} = 2\int_0^{\gamma}\int_0^{2\pi} \sin\theta\, d\theta\, d\varphi \cdot (3)^2 =$$

$$= 18\cdot 2\pi \left[-\cos\theta\right]_0^{\gamma} = 36\pi(1-\cos\gamma)$$

$$= 36\pi\left(1 - \frac{2\sqrt{2}}{3}\right)$$

flux :

$\vec{E} = \vec{r} \Rightarrow$

- $\vec{E}\cdot\vec{n} = 3$
- $\Phi = \iint_S \vec{E}\cdot\vec{n}\, dS = 3\cdot\text{aire} = 108\pi\left(1 - \frac{2\sqrt{2}}{3}\right)$

$$= 36\pi(3 - 2\sqrt{2}).$$